II.—THE PRINCIPLES OF DEMONSTRATIVE INDUCTION (II.).

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7. Laws of Correlated Variation of Determinates.—We have now completed our account of the arguments by which one attempts to establish laws of *Conjunction of Determinables*. Suppose that we have thus rendered it highly probable that $C_1 \ldots C_n$ is a S.S.C. of E, where E may itself be a complex characteristic of the form $E_1 \ldots E_m$. We now want to go further and to consider the connexion between various determinate values of $C_1 \ldots C_n$, on the one hand, and various determinate values of E, on the other. This is what Mr. Johnson seeks to formulate in his inductive methods. For this purpose we need some further postulates in addition to those which we used in the theory of necessary and sufficient conditions. We will begin by stating and commenting on these postulates.

Postulates.—(3) If C be a S.S.C. of E, and if there is at least one instance in which a certain determinate value *c* of C is accompanied by a certain determinate value *e* of E, then in *every* instance in which C has the value *c* E will have the value *e*. (We will call this *Postulate* 3, as we have already had two postulates.)

I will now make some comments on this postulate. (*a*) The converse of it is not assumed to hold. Our postulate states that *c* cannot be accompanied in some instances by *e* and in other instances by *e′*. But it does not deny that *e* may be accompanied in some cases by *c* and in others by *c′*. The point will be made clear by an example. Let E be the time of vibration of a compass-needle free to vibrate about its point of suspension in a magnetic field. Then the S.S.C. of E is a conjunction of three factors, *viz.*, the moment of inertia of the needle, its magnetic moment, and the intensity of the magnetic field. Call these three factors C_1, C_2, and C_3 respectively. Then the causal formula is in fact $E = 2\pi\sqrt{C_1/C_2C_3}$. It is plain that, if determinate values of C_1, C_2, and C_3 be taken, any repetition of them all will involve a repetition of the original value of E. But the original value of

E might occur when the values of C_1, C_2, and C_3 were different from their original values, provided the new values were suitably related among themselves.

(*b*) It will be noticed that the postulate is of the form required for the major premise of a demonstrative induction. For it is a hypothetical proposition in which the consequent is a universal categorical, and the antecedent is a particular categorical of the same quality and with the same subject and predicate as the consequent.

(*c*) In virtue of this postulate we can talk of *the* value of E which corresponds to a given value of C. But we cannot talk of *the* value of C which corresponds to a given value of E, since there may be several such values. Thus the postulate may be said to deny the possibility of a plurality of determinate total effects to a given determinate total cause, but to allow of a plurality of determinate total causes to a given determinate total effect. I propose to call this postulate the "Postulate of the *Uniqueness of the Determinate Total Effect.*"

(*d*) It must be clearly understood that, although in stating the postulate the single letters C and E have been used, they are meant to cover the case of conjunctions of factors, such as $C_1 \ldots C_n$ and $E_1 \ldots E_m$. In such cases the determinate *c* will represent the conjunction of a certain determinate value of C_1 with a certain determinate value of C_2 with . . . a certain determinate value of C_n. And similar remarks apply, *mutatis mutandis*, to *e*. Thus we shall have a different determinate value of C if we have a different determinate value of *at least one* of the determinables $C_1 \ldots C_n$, even though we have the same determinate values as before for all the other C-factors. And similar remarks apply, *mutatis mutandis*, to variations in the determinate value of E.

(4) This brings us to the fourth postulate. It runs as follows. If a total cause or a total effect be a conjunction of several determinables it is assumed that no determinate value of any of these factors either entails or excludes any determinate value of any of the other factors in this total cause or total effect. This may be called the "Postulate of *Variational Independence*". It should be compared with Postulate (1), which we called the postulate of *Conjunctive Independence.*

Now suppose that E is a conjunction of the determinables $E_1 \ldots E_m$. Let there be μ_1 determinates under E_1, μ_2 determinates under E_2, . . . and μ_m determinates under E_m. It follows from the Postulate of Variational Independence that the total number of different determinate values of E will be $\mu_1 \mu_2 \ldots \mu_m$. Let us call this the "*Range of Variation*" of E. Now it follows at once from the Postulate of the Uniqueness of the Determinate Total Effect that, if C be a S.S.C. of E, the range of variation of C cannot be narrower than the range of variation of E, though it may be wider. For to every different determinate value of E there must correspond a different determinate value of C, whilst several different determinate values of C may correspond to one and the same determinate value of E. Suppose that C is a conjunction of the determinables $C_1 \ldots C_n$. Let there be ν_1 determinates under C_1, ν_2 determinates under $C_2 \ldots$ and ν_n determinates under C_n. Then the range of variation of C is $\nu_1\nu_2 \ldots \nu_n$. And the principle which we have just proved is that $\nu_1\nu_2 \ldots \nu_n \geqslant \mu_1\mu_2 \ldots \mu_m$.

Now two different cases are possible. (*a*) Every determinable in E may have only a finite number of determinates under it. This alternative leads to nothing of great interest. (*b*) At least one of the determinables in E may have an infinite number of determinates under it. If so, the range of variation of E will be infinite. Consequently the range of variation of C must be infinite. But this will be secured if and only if at least one of the determinables in C has an infinite number of determinates under it. So we reach the general principle that if there is at least one factor in a total effect which has an infinite number of determinates under it then there must be at least one factor in any S.S.C. of this effect which has an infinite number of determinates under it.

We can now go rather further into detail by using the elements of Cantor's theory of transfinite cardinals. (*a*) Even if *all* the determinables in a total effect should have an infinite number of determinates under them it will be sufficient that *at least one* of the determinables in the total cause should have an infinite number of determinates under it. For the number of determinables in the total effect is assumed to be finite. Consequently the range of variation of the total effect will be an infinite cardinal raised to a finite power, even in the case supposed. Now it is known that any finite power of an infinite cardinal is equal to that infinite cardinal. Therefore it is enough, even in the case supposed, that at least one of the determinables in the total cause should have an infinite number of determinates under it. We can sum up our results in the form: "If *at least one* factor in the total effect has an infinite number of determinates under it it is *necessary* that at least one factor in the total cause should have an infinite number of determinates under it; and even if *all* the factors in the total effect have an infinite number of determinates under them it is *sufficient* that at least one of the factors in the total cause should have an infinite number of determinates under it ". (*b*) If the number of determinates under one of the determinables in E be infinite there are still two possible alternatives. In the first place the series of determinates may merely be " compact ", *i.e.*, it may merely be the case that there is a determinate of the series between any pair of determinates of the series. If so, it has the same cardinal number as the series of finite integers, *viz.* $\aleph_0$ the smallest of the transfinite cardinals. On the other hand, the series of determinates under this determinable may be " continuous " in the technical sense, as the points on a straight line are supposed to be. If so, it has the same cardinal number as the series of real numbers, *viz.*, $2^{\aleph_0}$. Now it is known that $2^{\aleph_0}$ is greater than $\aleph_0$. We can therefore enunciate the following general principle : " If any of the determinables in a total effect has under it a series of determinates which is strictly ' continuous ' then at least one of the determinables in the total cause must have under it a series of determinates which is not merely ' compact ' but is strictly ' continuous '."

Before leaving this subject there is one final question that might be raised. Is it possible that one or more of the determinables in a *total cause* should have an *infinite* number of determinates under it whilst all the determinables in the *total effect* have only a *finite* number of determinates under them ? There is certainly nothing in any of our postulates to rule out this possibility. It would be realised if, *e.g.*, the following state of affairs existed. Suppose that C is a total cause and that E is its total effect. Suppose that E has a finite number of determinate values e_1, e_2, etc. Suppose that the determinate values of C form a compact or a continuous series. And suppose finally that c_0 and every value of C between c_0 and c_1 determines the value e_1 of E, that c_1 and every value of C between c_1 and c_2 determines the value e_2 of E, and so on. I do not see anything impossible in a law of this kind, though I do not know of any quite convincing example of such laws. The following would be at least a plausible example. Suppose we take the three possible states of a chemical substance, such as water, *viz.*, the solid, the liquid, and the gaseous, as three determinates under a determinable. And suppose we say that this determinable is a total effect of which the two determinables of pressure and temperature constitute a total cause. Keep the pressure fixed at 76 cm. of mercury, and imagine the temperature to be varied continuously. Then every determinate value up to a temperature of zero on the centigrade scale determines the solid state, every determinate value from zero up to 100° determines the liquid state, and every determinate value above 100° determines the gaseous state.

I have said that the above is a *plausible* example of a case in which the same determinate total effect has an infinite plurality of different possible determinate total causes. But, when it is more carefully inspected, it can be seen not to be a *real* example. The fact is that we have not got here either the genuine total cause or the genuine total effect. The real total cause is a conjunction of three factors, *viz.*, the pressure, P, the total mass of the substance, M, and the quantity of heat contained in the substance, H. The real total effect is a conjunction of four factors; *viz.*, S, the amount of the substance in the solid state; L, the amount of the substance in the liquid state; G, the amount of the substance in the gaseous state; and T, the temperature of the substance. Our law of the conjunction of determinables is then that PMH is a S.S.C. of SLGT. Suppose that at the beginning of the experiment all the water is in the solid form, and is at a temperature below freezing-point. We will keep the determinate values of P and M constant throughout the experiment at the values p and m. And we will continuously increase H. At first L and G will have the values 0, and S will have the value m. As H is increased these values will remain constant, but T will continuously increase. This will go on till T reaches the melting-point of ice at the pressure p. If we now further increase H the values of S and L will begin to change continuously, whilst the value of T will remain at the melting-point of ice under the pressure p. The value of S will steadily diminish and that of L will steadily increase until we reach a stage at which the value of S is 0 and the value of L is m; *i.e.*, all the water will now be in the liquid state at the temperature of melting ice under the pressure p. If we still go on increasing the value of H the values of T will now start to increase steadily, and this will go on till the liquid water reaches the boiling-point under the given pressure. If H be still increased after this point we shall have the values of L and G changing, whilst T remains constant. This stage will go on as we increase H until all the water is converted into steam at the temperature of boiling water under the pressure p. At this stage S and L will have the values 0, whilst G will have the value m. If more heat be now put in, S, L, and G will henceforth keep constant at 0, 0, and m, respectively, and T will steadily rise.

We see then that at every stage *some* factor in the *total* effect is varying continuously as the factor H in the total cause varies continuously, although other factors in the total effect may at the same time be keeping constant in value. Thus the *total effect* changes continuously in value throughout the whole process, and to each determinate value of it there corresponds one and only one value of that factor in the total cause which is being continuously varied while the remaining cause-factors are kept constant. It is possible that, whenever it seems that a continuous set of different values of a total cause all determine the same value of a total effect, this is always due to our not having got the *total* cause and the *total* effect. But, although this may well be so, I do not see that there is any logical necessity that it should be so.

We come now to the remaining postulate of the correlated variation of determinates.

Before stating this postulate it will be convenient to introduce a certain notation which will enable us to formulate it briefly and clearly. Let us suppose that $C_1 \ldots C_n$ is a total cause of which E is the total effect. Consider a certain one factor in this total cause, *e.g.*, C_r. I propose to denote the conjunction of the remaining factors $C_1C_2 \ldots C_{r-1} C_{r+1} \ldots C_n$ by the single symbol Γ_{n-r}. The total cause can then be denoted by the symbol $C_r\Gamma_{n-r}$. Suppose now that a certain determinate value is assigned to each of the factors in Γ_{n-r}. We shall thus get a certain determinate value of Γ_{n-r}, and this may be denoted by γ^a_{n-r}. Let a certain determinate value of C_r be denoted by c^x_r. Then the determinate value of the total cause may be denoted by $c^x_r\gamma^a_{n-r}$. To this there will correspond a certain one determinate value of E. Let us denote this by $e^{x,a}_{r,n-r}$. We are now in a position to state our postulate.

(5) Let $C_1 \ldots C_n$ be a total cause of which E is the total effect. Select any one factor C_r from this, and assign to the remainder Γ_{n-r} any fixed value γ^a_{n-r}. Then, if there are *at least two* values of C_r, *e.g.*, c^x_r and c^y_r, which determine *different* values of E, *every* different value of C_r in combination with γ^a_{n-r} will determine a *different* value of E.

With the notation explained above the postulate can be stated very simply in the symbolism of *Principia Mathematica*. It will run as follows :—

$$(\exists x, y) \, . \, c^x_r \neq c^y_r \, . \, e^{x,a}_{r,n-r} \neq e^{y,a}_{r,n-r} : \supset_{a,r} : c^x_r \neq c^y_r \, . \supset_{x,y} . \, e^{x,a}_{r,n-r} \neq e^{y,a}_{r,n-r}.$$

Now there are two other propositions which are logically equivalent to this postulate. The first is reached by taking the contra-positive of Postulate 5. We will call it (5*a*). It runs as follows :—

(5*a*) Let $C_1 \ldots C_n$ be a total cause of which E is the total effect. Select any one factor C_r from this, and assign to the remainder Γ_{n-r} any fixed value γ^a_{n-r}. Then, if there are *at least* *two* values of C_r, *e.g.*, c_r^x and c_r^y, which in combination with γ_{n-r}^a determine *the same* value of E, *every* value of C_r in combination with γ_{n-r}^a will determine *the same* value of E.

This can be put in the symbolism of *Principia Mathematica* as follows :—

$$(\exists x, y) \,.\, c_r^x \neq c_r^y \,.\, e_{r, n-r}^{x, a} = e_{r, n-r}^{y, a} : \supset_{a, r} : (x, y) \,.\, e_{r, n-r}^{x, a} = e_{r, n-r}^{y, a}.$$

The second logically equivalent form of Postulate 5 may be called (5*b*). It is reached by substituting for the original hypothetical proposition the equivalent denial of a certain conjunctive proposition, in accordance with the general principle that " if p then q " is equivalent to the denial of the conjunction " p and not-q ". It runs as follows :—

(5*b*) Let $C_1 \ldots C_n$ be a total cause of which E is the total effect. Select any one factor C_r from this, and assign to the remainder Γ_{n-r} any fixed value γ_{n-r}^a. Then it cannot be the case *both* that there is a pair of values of C_r which in combination with γ_{n-r}^a determine *different* values of E, *and also* that there is a pair of values of C_r which in combination with γ_{n-r}^a determine *the same* value of E. This can be symbolised as follows :—

$$\sim\{(\exists x, y) \,.\, c_r^x \neq c_r^y \,.\, e_{r, n-r}^{x, a} \neq e_{r, n-r}^{y, a} : (\exists x, y) \,.\, c_r^x \neq c_r^y \,.\, e_{r, n-r}^{x, a} = e_{r, n-r}^{y, a}\}.$$

I will now make some comments on this postulate. (*a*) It will be seen, on referring back to the first section of this paper, that (5) and (5*a*) are propositions of the form required to enable them to be used as major premises in demonstrative inductions. They are used as such by Mr. Johnson in his " *Figure of Difference* " and his " *Figure of Agreement* " respectively.

(*b*) It will be noticed that, when the conditions of (5) are fulfilled, not only is the *presence* of C_r relevant to the *presence* of E, but also the *variations* of C_r are relevant to the *variations* of E. Postulate 5 may therefore be called the " Postulate of *Variational Relevance* ". When the postulate is put in the equivalent form (5*a*), and the conditions are fulfilled, the presence of C_r is relevant to the presence of E, but the variations of C_r are irrelevant to the variations of E. So, in this form, it may be called the " Postulate of *Variational Irrelevance* ". An interesting example of variational irrelevance is furnished by Prof. H. B. Baker's discovery that gases which normally combine with explosive violence when a spark is passed through a mixture of them will not combine at all if they be completely dry. Thus the presence of *some* water is a necessary condition for any combination to take place in the assigned circumstances. But, granted that there is some water present, no difference in the amount of it seems to make any appreciable difference to the completeness or the violence of the combination which takes place when a spark is passed through the mixed gases.

(*c*) It must of course be clearly understood that, when the conditions of (5) are fulfilled, it follows only that variations of C_r are relevant so long as Γ_{n-r} is kept fixed *at the value* γ^a_{n-r}. For other values of Γ_{n-r} variations in C_r might be irrelevant. Similarly, when the conditions of (5*a*) are fulfilled, it follows only that variations in C_r are irrelevant so long as Γ_{n-r} is kept fixed *at the value* γ^a_{n-r}. For other values of Γ_{n-r} variations in C_r might be relevant.

(*d*) Finally we come to the question: "Is this postulate true?" It seems to me quite certain that it is not. The fact is that Mr. Johnson, who first stated it, has altogether ignored the possibility of natural laws which take the form of periodic functions. Suppose there were a natural law of the form $E = C_1 \sin C_2$. Let C_1 be assigned a certain value. Take any value C_2^x of C_2. Then, for every value of C_2 that differs from this by an integral multiple of 2π, E will have the same value. On the other hand, for every value of C_2 which does not differ from this by an integral multiple of 2π, E will have a different value. Thus (5*b*) is directly contradicted. Nor is the kind of law which leads to these results at all fanciful. Such laws hold in electro-magnetism for alternating currents and the magnetic forces which depend on them. Thus the effect of the Postulate is to exclude all laws which take the form of periodic functions. And there is no *a priori* objection to such laws, whilst some important natural phenomena are in fact governed by laws of this kind.

It is worth while to remark that the existence of periodic laws answers in the affirmative a question which was raised and left unanswered in our comments on Postulate (4). The question was whether it is possible that a single determinate value of a total effect should correspond to an infinite plurality of alternative values of the total cause. In the case of periodic laws this possibility is realised. In our example, if C_1 be fixed, every one of the infinite class of values of C_2 which differ from each other by an integral multiple of 2π will determine one and the same value of E.

(8) Mr. Johnson's "Figures of Induction". It only remains to explain and exemplify Mr. Johnson's "Figures of Induction". These are based on Postulate (3), *i.e.*, the Postulate of the Uniqueness of the Determinate Total Effect, and on one

form or other of Postulate (5). The "Figure of Difference" uses this postulate in its first form, *i.e.*, in the form of the Postulate of Variational Relevance. The "Figure of Agreement" uses it in the second form (5*a*), *i.e.*, in the form of the Postulate of Variational Irrelevance. All the Figures also presuppose Postulate 4, *i.e.*, the Postulate of Variational Independence. And, since they all presuppose that a certain set of determinables has been shown to stand in the relation of total cause to a certain other set of determinables as total effect, they all presuppose the two postulates of Conjunctive Independence and of Smallest Sufficient Conditions. For these are involved in the arguments which are used in establishing laws of the Conjunction of Determinables. We will now consider the Figures in turn.

(i) *Figure of Difference.*—The premises are as follows :—

$C_1 \ldots C_n$ is a total cause of which E is the total effect. (*a*).

In a certain instance a certain determinate value $C_r^u \gamma_{n-r}^a$ is accompanied by a certain determinate value *e* of E. (*b*).

In a certain instance a certain determinate value $c_r^v \gamma_{n-r}^a$ is accompanied by a certain determinate value *e′* of E. (*c*).

c_r^u and c_r^v are different values of C_r; and *e* and *e′* are different values of E. (*d*).

The argument runs as follows :—

From (*a*), (*b*), and Postulate (3) it follows that *every* instance of $c_r^u \gamma_{n-r}^a$ is also an instance of *e*.

From (*a*), (*c*), and Postulate (3) it follows that *every* instance of $c_r^v \gamma_{n-r}^a$ is also an instance of *e′*.

From these conclusions, together with (*d*) and Postulate (5), the following conclusion results: "Corresponding to *each* value of $C_r \gamma_{n-r}^a$ there is a certain value of E, such that *every* instance of that value of $C_r \gamma_{n-r}^a$ is an instance of that value of E. And for *every* different value of $C_r \gamma_{n-r}^a$ the corresponding value of E is different." That is

$$c_r^x \neq c_r^y \,.\supset_{x,\,y} .\, e_{r,\,n-r}^{x,\,a} \neq e_{r,\,n-r}^{y,\,a}.$$

(ii) *Figure of Agreement.*—The premises are as follows :—

$C_1 \ldots C_n$ is a total cause of which E is the total effect. (*a*).

In a certain instance a certain determinate value $c_r^u \gamma_{n-r}^a$ is accompanied by a certain determinate value *e* of E. (*b*).

In a certain instance a certain determinate value $c_r^v \gamma_{n-r}^a$ is accompanied by the same determinate value *e* of E. (*c*).

c_r^u and c_r^v are different values of C_r. (*d*).

The argument runs as follows :—

From (*a*), (*b*), and Postulate (3) it follows that *every* instance of $c_r^u \gamma_{n-r}^a$ is also an instance of *e*.

From (*a*), (*c*), and Postulate (3) it follows that *every* instance of $c_r^v \gamma_{n-r}^a$ is also an instance of *e*.

From these conclusions, together with (*d*) and Postulate (5*a*), the following conclusion results : " Corresponding to *each* value of $C_r \gamma_{n-r}^a$ there is a certain value of E, such that *every* instance of that value of $C_r \gamma_{n-r}^a$ is an instance of that value of E. And for *every* value of $C_r \gamma_{n-r}^a$ the corresponding value of E is the same, *viz.*, *e*." That is

$$(x, y) \, . \, e_{r, n-r}^{x, a} = e_{r, n-r}^{y, a} = e.$$

I will now make some comments on these two figures. The important point to notice is that each makes a *double* generalisation by means of two different applications of demonstrative induction. The first generalises from a given *instance* of a given value to *all instances* of *that* value. This part of the argument rests on the Postulate of the Uniqueness of Determinate Total Effects. The second generalises from a given *pair of values* of a certain determinable cause-factor to *every pair of values* of that cause-factor. This part of the argument rests on the Postulate of Variational Relevance or Variational Irrelevance. The final result sums up both generalisations.

It may be remarked that, when we have the premises needed for the Figure of Agreement, we can reach a more determinate conclusion than when we have the premises needed for the Figure of Difference. In the former case we know the determinate value of E which will be present in every instance in which any value of C_r is combined with γ_{n-r}^a. In the latter case we know only that a different determinate value of E will be present for each different determinate value of C_r combined with γ_{n-r}^a. We do not know what value of E will be correlated with each different value of $C_r \gamma_{n-r}^a$. To discover this we need to use the methods of *Functional Induction ;* and this is a branch of *Problematic*, not of *Demonstrative*, Induction, and so falls outside the scope of this paper. Thus any complete inductive investigation begins and ends with Problematic Induction, and uses Demonstrative Induction only in its intermediate stages. It begins with Problematic Induction in order to establish Laws of the Conjunction of Determinables. In order to get these into the form of laws which express the relation of total cause to total effect it has to use the kind of deductive arguments which we considered in connexion with Necessary and Sufficient Conditions. In order to discover which factors in the total cause are variationally relevant and which are variationally irrelevant it has to use Mr. Johnson's figures, or something equivalent to them. And, in order to discover the detailed functional relation between variations in the total cause and variations in the total effect, it has finally to resort to a form of Problematic Induction.

(iii) *Figure of Composition.*—The premises are as follows :—

$C_1 \ldots C_n$ is a total cause of which E is the total effect. (*a*).

In a certain instance a certain determinate value $C_r^u \gamma_{n-r}^a$ is accompanied by a certain determinate value *e* of E. (*b*).

In a certain instance a certain determinate value $c_r^v \gamma_{n-r}^a$ is accompanied by a certain determinate value *e′* of E. (*c*).

In a certain instance a certain determinate value $c_r^w \gamma_{n-r}^b$ is accompanied by a certain determinate value *e* of E. (*d*).

The three values of C_r are all different, and *e′* is different from *e*. (*e*).

The argument runs as follows :—

From (*a*), (*b*), and Postulate (3) it follows that *every* instance of $c_r^u \gamma_{n-r}^a$ is also an instance of *e*.

From (*a*), (*c*), and Postulate (3) it follows that *every* instance of $c_r^v \gamma_{n-r}^a$ is also an instance of *e′*.

From (*a*), (*d*), and Postulate (3) it follows that *every* instance of $c_r^w \gamma_{n-r}^b$ is also an instance of *e*.

Now it is impossible that γ_{n-r}^a should be the same as γ_{n-r}^b. For, if we first take (*b*) and (*d*) together, and then take either (*b*) and (*c*) together or (*c*) and (*d*) together, this would directly contradict Postulate (5*b*), in view of (*e*).

The final conclusion which results is this : " Corresponding to *every* different value of C_r which, in conjunction with some value of Γ_{n-r}, determines the same value *e* of E there is a different value of Γ_{n-r}. And, in *every* instance in which a certain value of C_r is present along with *e* and some value of Γ_{n-r}, Γ_{n-r} will be present in the value that corresponds to this value of C_r."

I must remark that Mr. Johnson's formulation of this figure at the bottom of page 225 of Part II. of his *Logic* seems to me quite unsatisfactory. He there mentions only two instantial premises, whilst it is quite evident from the verbal statement of the figure which he makes earlier on the same page that a third instantial premise is essential to distinguish this from the Figure of Difference.

We can carry the argument a step further if we now add the premise (f) that γ^a_{n-r} and γ^b_{n-r} are known to differ *only* in the respect that a certain determinable C_s is present in the former in the value c^a_s and in the latter in the value c^b_s. In that case γ^a_{n-r} can be written as $c^a_s\gamma^\alpha_{n-r-s}$, and γ^b_{n-r} can be written as $c^b_s\gamma^\alpha_{n-r-s}$.

The final conclusion then runs as follows: "Corresponding to *every* different value of C_r which, in conjunction with γ^α_{n-r-s} and with some value of C_s, determines the same value e of E there is a different value of C_s. And, in *every* instance in which a certain value of C_r is present along with e and some value of C_s, C_s will be present in the value which corresponds to this value of C_r."

We may symbolise the value of E which is always present when $c^x_r c^a_s \gamma^\alpha_{n-r-s}$ is present by $e^{x, a, \alpha}_{r, s, n-r-s}$. The first clause of the above conclusion can then be symbolised as follows:—

$$c^x_r \neq c^y_r \,.\, e^{x, a, \alpha}_{r, s, n-r-s} = e^{y, b, \alpha}_{r, s, n-r-s} = e : \supset_{x, y} \,.\, c^a_s \neq c^b_s.$$

The first clause of the conclusion which we reached before adding the premise (f) may be symbolised as follows:—

$$c^x_r \neq c^y_r \,.\, e^{x, a}_{r, n-r} = e^{y, b}_{r, n-r} = e : \supset_{x, y} \,.\, \gamma^a_{n-r} \neq \gamma^b_{n-r}.$$

(iv) *Figure of Resolution.*—This figure is in a different position from the others. Here we have three observations which directly conflict with Postulate (5b) given the premise that $C_1 \ldots C_n$ is a total cause of which E is the total effect. It is evident that, in such a case, the only solution is to suppose that we were mistaken in believing that every factor in $C_1 \ldots C_n$ is simple. One at least of them must be a conjunction of at least two determinables, *e.g.*, K_1 and K_2. The kind of premises which would lead to this conclusion are the following:—

In a certain instance a certain determinate value $C^u_r\gamma^a_{n-r}$ is accompanied by a certain determinate value e of E. (b).

In a certain instance a certain determinate value $C^v_r\gamma^a_{n-r}$ is accompanied by a certain determinate value e' of E. (c).

In a certain instance a certain determinate value $C^w_r\gamma^a_{n-r}$ is accompanied by a certain determinate value e of E. (d).

c^u_r, c^v_r, and c^w_r are all different. And e' is different from e. (e).

It is evident that, if (b) and (d) be taken together, and if either (b) and (c) or (c) and (d) be taken together, there is a direct conflict with Postulate (5b). The only solution is to suppose that C_r is really a conjunction of two determinables, K_1 and K_2. In that case c^u_r may be k_1k_2, c^v_r may be $k_1'k_2'$, and c^w_r may be $k_1''k_2''$; and the contradiction will be avoided. It seems needless to pursue this into further detail.

Complete Symbolic Statement of the First Three Figures.—I will bring this paper to an end by giving a complete symbolisation for the premises and the conclusions of the first three of Mr. Johnson's figures. Mr. Johnson's own symbolism seems to me to be very inadequate. For the present purpose we shall need two further bits of symbolism. (i) I will symbolise the premise that $C_1 \ldots C_n$ is a total cause of which E is the total effect by $C_1 \ldots C_n \rightarrow E$. (ii) We need a symbol for the statement that x is an instance of a conjunction of characteristics, A, B, C . . . Z. I shall denote this by $[AB \ldots Z]x$. We are now in a position to deal with the figures of Difference, Agreement, and Composition.

Difference.

$$C_1 \ldots C_n \rightarrow E \quad (a)$$
$$[c_r^u \gamma_{n-r}^a e]p \quad (b)$$
$$\therefore [c_r^u \gamma_{n-r}^a]\xi \supset_\xi [e]\xi \quad \text{(by Postulate 3)}$$
$$[c_r^v \gamma_{n-r}^a e']q \quad (c)$$
$$\therefore [c_r^v \gamma_{n-r}^a]\xi \supset_\xi [e']\xi \quad \text{(by Postulate 3)}$$
$$c_r^u \neq c_r^v . e \neq e' \quad (d)$$
$$\therefore c_r^x \neq c_r^y . \supset_{x,y} . e_{r,n-r}^{x,a} \neq e_{r,n-r}^{y,a} : [c_r^x \gamma_{n-r}^a]\xi \supset_{\xi,x} [e_{r,n-r}^{x,a}]\xi \quad \text{(by Postulate 5).}$$

Agreement.

$$C_1 \ldots C_n \rightarrow E \quad (a)$$
$$[c_r^u \gamma_{n-r}^a e]p \quad (b)$$
$$\therefore [c_r^u \gamma_{n-r}^a]\xi \supset_\xi [e]\xi \quad \text{(by Postulate 3)}$$
$$[c_r^v \gamma_{n-r}^a e]q \quad (c)$$
$$\therefore [c_r^v \gamma_{n-r}^a]\xi \supset_\xi [e]\xi \quad \text{(by Postulate 3)}$$
$$c_r^u \neq c_r^v \quad (d)$$
$$\therefore (x, y) . e_{r,n-r}^{x,a} = e_{r,n-r}^{y,a} = e : [c_r^x \gamma_{n-r}^a]\xi \supset_\xi [e]\xi \quad \text{(by Postulate 5a).}$$

Composition.

$$C_1 \ldots C_n \rightarrow E \quad (a)$$
$$[c_r^u \gamma_{n-r}^a e]p \quad (b)$$
$$\therefore [c_r^u \gamma_{n-r}^a]\xi \supset_\xi [e]\xi \quad \text{(by Postulate 3)}$$
$$[c_r^v \gamma_{n-r}^a e']q \quad (c)$$
$$\therefore [c_r^v \gamma_{n-r}^a]\xi \supset_\xi [e']\xi \quad \text{(by Postulate 3)}$$
$$[c_r^w \gamma_{n-r}^b e]t \quad (d)$$

$\therefore\ [c_r^w \gamma_{n-r}^b]\xi \supset_\xi [e]\xi$ (by Postulate 3)

$c_r^u \neq c_r^v \,.\, c_r^v \neq c_r^w \,.\, c_r^w \neq c_r^u \,.\, e \neq e'$ (e)

$\therefore\ c_r^x \neq c_r^y \,.\, e_{r,\,n-r}^{x,\,a} = e_{r,\,n-r}^{y,\,b} = e : \supset_{x,\,y} . \gamma_{n-r}^a \neq \gamma_{n-r}^b$

(by Postulate 5*b*)

Denote the value of Γ_{n-r} which corresponds to c_r^x by ${}_r^x\gamma_{n-r}^e$.

Then $[c_r^x \Gamma_{n-r} e]\xi \supset_\xi [{}_r^x\gamma_{n-r}^e]\xi$.

Let $\Gamma_{n-r} = \mathrm{C}_s \gamma_{n-r-s}^\alpha$ (f).

Then $c_r^x \neq c_r^y \,.\, e_{r,\,s,\,n-r-s}^{x,\,a,\,\alpha} = e_{r,\,s,\,n-r-s}^{y,\,b,\,\alpha} = e : \supset_{x,\,y} c_s^a \neq c_s^b$.

Denote the value of C_s which corresponds to c_r^x by ${}_r^x c_s^e$.

Then ${}_r^x\gamma_{n-r}^e = {}_r^x c_s^e \gamma_{n-r-s}^\alpha$.

Whence $[c_r^x \mathrm{C}_s \gamma_{n-r-s}^\alpha e]\xi \supset_{\xi,\,x} [{}_r^x c_s^e \gamma_{n-r-s}^\alpha]\xi$

$\supset_{\xi,\,x} [{}_r^x c_s^e]\xi$.

Thus the complete final conclusion is :—

$$c_r^x \neq c_r^y \,.\, e_{r,\,s,\,n-r-s}^{x,\,a,\,\alpha} = e_{r,\,s,\,n-r-s}^{y,\,b,\,\alpha} = e : \supset_{x,\,y} c_s^a \neq c_s^b :. [c_r^x \mathrm{C}_s \gamma_{n-r-s}^\alpha e]\xi \supset_{\xi,\,x} [{}_r^x c_s^e]\xi.$$